

MATHEMATICS IN ITS MAKING .

1. PRELIMINARY REMARKS

With this brief idea of brick technology in Indian history, we now pass on to the problem of the making of mathematics in ancient India. The main points we are going to argue are as follows :

1. A class of texts come down to us with the general title *Sulva-sutra-s* (spelled also as *Sulba-sutra-s*) which are for us the earliest codified documents for the making of mathematics, specially geometry, in India.

2. The actual mathematical knowledge contained in these, judged specially in the ancient context, cannot but be considered as remarkable.

3. Notwithstanding the usual assumption that this mathematics was created by the Vedic priests, the internal evidences of the texts indicate that it was the direct outcome of the theoretical requirements mainly of the brick-makers and brick-layers, who were using burnt bricks and whose status within the general framework of the social norm of the Vedic priests is at best questionable.

4. With all the uncertainties and controversies about the actual date of these texts, there is no possibility of placing these outside the period intervening the two urbanizations—the “Dark Period” or “Dark Age” of our archaeologists, one of the conspicuous features of which is the loss of the sophisticated tradition of brick technology of the First urbanization.

5. Archaeologically, therefore, we are confronted here with an apparent anomaly. Mathematics came into being in ancient India to meet the theoretical requirements primarily of brick technology in a period in which there is no brick technology—not at least in any notable scale, specially in the sense of the technology of making and using burnt bricks which is presupposed by the *Sulva-sutra-s*.

6. This anomaly can perhaps be resolved by the presumption that the mathematics—though codified much later in the form of the *Sulva-sutra-s*—was actually the creation of the

First Urbanization, in spite of the uncertainty or gap at the present stage of our knowledge of the mode of its transmission to the later period.

7. Though we do not have any direct document embodying mathematical knowledge in the First Urbanization, there are strong circumstantial evidences in favour of the assumption just mentioned. In view of this, it is not easy to dismiss Gordon Childe's postulate of the unknown perishable materials in which this mathematics could have been codified in the Indus Valley Civilization—a postulate on the basis of which he wanted to trace the roots of classical Indian Science to the achievements of the Harappan culture.

The starting point of these arguments, however, is a widely current misconception about the making of mathematics in general.

2. ORIGIN OF GEOMETRY: HERODOTUS AND RECENT CORRECTIONS OF HIS VIEW

According to Herodotus, the Greeks first received the knowledge of geometry from the Egyptians who, in their turn, developed it from the practice of land-measurement required for administrative purposes. Here is the observation of Herodotus,¹ which has found place in most of the histories of science :

Sesostris also, they declared, made a division of the soil of Egypt among the inhabitants, assigning square plots of ground of equal size to all, and obtaining his chief revenue from the rent which the holders were required to pay him every year. If the river carried away any portion of a man's lot, he appeared before the king, and related what had happened ; upon which the king sent persons to examine, and determine by measurement the exact extent of the loss ; and thenceforth only such a rent was demanded of him as was proportionate to the reduced size of his land. From this practice, I think, geometry first came to be known in Egypt, whence it passed into Greece. The sundial, however, and the gnomon, with the division of the day into twelve parts, were received by the Greeks from the Babylonians.

Depending on this, it is often assumed that the science of geometry developed from the practical requirements of land measurement. As C. Singer puts it, "The development of rights in land demanded some sort of surveying. Greek tradition has

1. Quoted by Smith I.51-52.

it that the inundation of the Nile, by obliterating all landmarks, forced on the Egyptians an annual re-measurement of their fields. Thus *geo-metry* (literally 'earth-measurement') was born".²

But such a view is not fully endorsed by archaeology. It is true that the economic and administrative requirements of the urban revolution in Egypt—and also in Mesopotamia—did call forth some geometrical knowledge connected with land measurement. But it was enough for the purpose to have on the whole an approximate knowledge, and not geometry in the sense of an exact science. As Gordon Childe³ puts it :

The conditions of urban economy . . required some knowledge of geometrical relations. The areas of fields must be determined for estimations of the seed required for sowing them and the rent or tax that might be exacted in respect of them. But for such estimates and assessments absolute accuracy was unnecessary : the bailiff only wanted to know roughly how much grain to allow for each field ; the tax-collector needed a general idea of the yield to be expected. We have seen that even before 3000 B.C. the Sumerians were calculating the areas of fields as the product of length by breadth ; they were, that is, applying the correct geometrical formula for the area of a rectangle. In later documents the areas of irregular quadrangles are calculated by various approximations, usually the mean of the products of the two pairs of adjacent sides. Polygonal fields were divided up into quadrangles and triangles, the areas of which were similarly calculated. In Egypt, even in New Kingdom contracts, the area of a four-sided field is taken as half the sum of two adjacent sides multiplied by half the sum of the remaining sides. In the case of a triangular field, the length of two sides were added together and halved, and then multiplied by half the length of the third side. The documents just examined generally contain plans of the fields in question. The lengths are written in along the sides, but the plans are not drawn accurately or to scale. *The theory that exact geometry arose out of land-hurveying in Egypt or Babylonia is not supported by the evidence at our disposal.*

Where, then, are we to look for the making of geometry as exact science in ancient Egypt and Mesopotamia? Gordon Childe wants us to look for this in the techniques of the masons, architects and engineers. As he puts it, "On the other hand [i.e. as contrasted with the land-surveying administrators], architects and engineers often required more exact calculations to

2. Singer 4.

3. Childe MMH 205-7.

fulfil the tasks imposed on them. The accuracy of a pyramid was a matter of ritual significance. To secure it, the sizes of the blocks facing it must be accurately calculated".⁴

The pyramids were made, of course, of blocks of stone rather than bricks. But J.D. Bernal⁵ very explicitly discusses the question of the making of mathematics as connected with brick technology. We quote him at some length.

The operation of building itself also contributed, probably even before land survey, to the foundation of geometry. Originally, town buildings were simply village huts made of wood or reeds. In cities, with a restricted space and danger of fire houses of pise or rammed mud were a great improvement. The next step was to have even greater consequences: the invention of the standard moulded block of dried mud—the brick. The brick may not be an original invention, but a copy, in the only material available in the valley country, of the stone slabs that came naturally to hand for dry walling in the hills. Bricks are difficult to fit together unless they are rectangular, and their use led necessarily to the idea of the right angle and the use of the straight line—originally the stretched line of the cordmaker or weaver.

The practice of building in brick, particularly of large religious buildings of pyramid form, gave rise not only to geometry, but also to the conception of areas and volumes of figures and solids reckonable in terms of the lengths of their sides. At first only the volume of rectangular blocks could be estimated, but the structural need for tapering or buttering a wall led to more complicated shapes like that of the pyramid. The calculation of the volume of a pyramid was the highest flight of Egyptian mathematics and foreshadowed the methods of the integral calculus.

Also from building came the practice of the plan to scale. Such a plan for a town together with the architect's rule is for instance shown in the status of Gudea of Lagash in c. 2250 B.C. With these mathematical methods an administrator was able to plan the whole operation of brick or stone building in advance. He could estimate accurately the number of labourers wanted, the amount of materials and food they would need, and the time the job would take. The techniques were readily extendable from the city to the country in the lay-out of fields, the calculation of their areas, and the estimate of their yields for revenue purposes. This is the origin of mapping and surveying. It was this practical use that later gave rise to the term of geometry—land measurement. Mathematics, indeed, arose in the first place as an auxiliary method of production made necessary and possible by city life.

4. *Ibid* 207.

5. Bernal 121.

3. R. S. SHARMA AND SULVA GEOMETRY

Notwithstanding etymology (*geo-metry* = *earth-measurement*); the theory of geometry, i.e. in the sense of exact science, emerging from the practice of land-measurement for administrative purposes is thus no longer tenable. With this point in mind, let us return to the Indian evidences.

We have no direct evidence, of course, of any practice of land-measurement in the Indus Civilization. However, judging from the circumstance that this civilization thrived on agricultural surplus collected from a vast area, one is inclined to presume that some such practice was prevalent also in the Indus Civilization. The presumption seems to be supported by the analogy of the two other primary centres of urban revolution, namely Egypt and Mesopotamia. The analogy of Egypt and Mesopotamia, again, would lead us to think that the administrative requirements in the Indus Civilization could have been satisfied only with some approximate knowledge of the area of the land surveyed rather than geometry as an exact science.

Some practice of measuring land with rods or poles is perhaps suggested—though desultorily—by a few verses of the *Rgveda*.⁶ But it is difficult to be exact about what is really implied by such archaic references, specially in view of the fact that in the Rgvedic period wealth was basically conceived in terms of cattle rather than land or agricultural products. In any case, there is no hint whatsoever either in the Rgvedic verses or even in Sayana's commentary on these of anything that may be taken as directly or indirectly indicating any geometrical interest of the early Vedic poets.

The first references to land-measurement for administrative purposes are to be found in the Pali sources, which belong to the earlier period of Second Urbanization. Since the rope or *rajju* was used for these measuring purpose, the officer entrusted with the work was called *rajju-gahaka-amacca*, or per-

6. *RV* i.100.18; i.110.5 and iii.38.3. See Srinivasan 7. It is usually assumed that the early Rgvedic people were pastoral nomads, which if true would hardly require of them the technique of land measurement. The earliest reference in Vedic literature to what could perhaps been mathematics is to be found in *Ch. Up* vii. 2, where the word *rasi* is interpreted by Sankara as mathematics.

haps simply as *rajjuka*, literally "the rope-holder". As B. B. Datta observes, "In the Pali literature, we find the terms *rajjuka* or *rajju-gahaka* (rope-holder) for the king's land-surveyor. The first of these terms appears copiously... in the inscriptions of Emperor Asoka".⁷

R. S. Sharma, while discussing the tithe system of the period, naturally goes into some detail of the evidence⁸:

The two references (in the *Jatakas*) which relate to the measuring of field by royal officers, are capable of being interpreted in a way which may suggest some sort of 'ground rent'. Buhler compares the *rajjugahaka-amacca* with the Land Revenue Settlement Officer of British India and suggests that measurement was used for assessing ground rent'. But Fick surmises that land was measured either to form an approximate idea of the amount of rent payable by the subjects to the king or to determine the average produce to be brought to the king's store-room. Nevertheless, the fact that in measuring the field the *rajjugahaka-amacca* was conscious of doing nothing which might cause loss either to the *raja* or to the *khettsamika* ('the owner of the field') or *kutumba* ('the royal property') lends strength to Buhler's hypothesis that the land was measured for the purpose of levying rent on it.

Because it is outside the scope of R. S. Sharma's present theme of discussion, he does not raise in this connection the question of the making of geometry from this practice of land-measurement. Nevertheless, it is relevant for our purpose to note in this connection that the two references to the *rajjugahaka-amacca* or 'rope-holding officer' found in the Pali *Jataka*-s do not mention geometry. As a matter of fact, in the entire voluminous literature called the *Jataka*-s we have no hint of geometry anywhere. The repetition of the stereo-typed phrase of "accomplishment in the eighteen branches of learning", which we come across in this literature,⁹ is of course, not explained in the *Jataka*-s, though from other Pali sources we are left to presume that these refer to divination, auguries, interpretation of prognostics, etc.¹⁰ which had some sort of prestige for the popular minds in those days, though, the Buddha himself is supposed to have disapproved of these as undesirable

7. B. B. Datta SS 9.

8. R. S. Sharma in SSP I.i. 7-8.

9. Cowell *Jatakas* i. 126, 203, 285; ii. 60, 168, 287; iv. 104, 105.

10. See, e.g., The *Mahasilam*, Tr., SBE. XI, 196 ff.

branches of learning. In any case, in spite of the mention of the practice of land-measurement in the period, there is no evidence to show that geometry as an exact science emerged from this practice.

At the same time, there seems to be a rather tricky point in this connection. The earliest Indian documents embodying geometry as exact science, as we have already said, are called the *Sulva-sutra-s* and the word *sulva* (or *sulba*), to which this body of literature owes its name, does mean the "rope". It is in fact an elitist-esoteric equivalent for the word *rajju*. Since the word *rajju* is used in connection with land-measurement—and perhaps largely under the older hypothesis that geometry originated in Egypt from the practice of land-measurement—R.S. Sharma in his *Material Culture and Social Formations in Ancient India* ventures the hypothesis that this practice of land-measurement by rope could have originally been at the basis of the *Sulva* geometry. As he puts it, "For fixing individual possession of fields and assessing taxes knowledge of measurement was necessary. Methods of calculating the areas of the circle, rectangle, etc., or the method of converting circles into squares, though prescribed in the religious context in the *Sulvasutra-s*, may have arisen in response to the needs of field agriculture".¹¹

The hypothesis seems to be a hasty one. What is embodied in the *Sulva-sutra-s*, as we are going to see, is geometry as an exact science—much more than the approximate calculations of areas etc. which, as Gordon Childe has shown, can be the outcome of the early administrative requirements of field-measurement. Secondly, though the texts derive their name from the rope or string—*sulva*—and though use of the string or the rope is of much importance for the making of geometry in these texts, the use of the rope in the *Sulva-sutra-s* has little to do with field-measurement, beyond perhaps the measurement of the ground-plan of certain brick structures. The construction of these bricks structures form, in a sense, the be all and end all of the *Sulva-sutra-s* and the use of the string or rope is concerned above all with the techniques of making or laying these bricks. The technological context of the use of "rope", in short, is different altogether. It is the context in

11. R. S. Sharma MCSFAI 108.

which the string or rope is still found to be used by the masons and architects. In fact, if it is at all conceivable to scrap the brick technology from the *Sulva* texts—which, as we shall see, is really not conceivable—practically nothing that can be called geometry—or more broadly mathematics—is at all left in the *Sulva-sutra-s*. This consideration, by itself, seems to make R.S. Sharma's hypothesis unacceptable. Besides the general consideration remains that nowhere in Indian literature we come across even any remote suggestion connecting the administrative requirements of field measurement with the making of geometry even as an approximate knowledge, not to speak of geometry as an accurate science, which we have in the *Sulva* texts.

At the same time, all this leads us to a situation which, from the archaeological viewpoint at any rate, is apparently anomalous. The *Sulva* texts belong to a period which had no brick technology. Before passing on to discuss this, let us have a preliminary idea of how the *Sulva* geometry is inconceivable without brick technology.

4. BRICK TECHNOLOGY AND MAGICO-RELIGIOUS BELIEFS

The main theme of the *Sulva-sutra-s* is the construction of certain brick-structures. The structures range from comparatively simpler to highly complicated ones and the texts concentrate mainly on the latter. Without developing a body of mathematical knowledge it was not possible to be accurate about the constructions. From this point of view, it was necessary to develop the mathematical—particularly geometrical—knowledge, and it is this that we read in the *Sulva* texts.

What complicates matters, however, is a different point. The brick-structures were supposed to be not just brick structures constructed for secular purposes. These were called *citi-s* or *agni-s*—meaning altars required for the performance of Vedic sacrifices and the sacrifices were intended to have magico-religious efficacy. Hence, apparently at any rate, the question of the making of mathematics remains in the *Sulva* texts as somehow interlinked with a body of magico-religious system.

In such a circumstance, it is first of all necessary to be clear about one point. How far is the body of the magico-religious beliefs internally connected with the technological

questions concerning the brick-constructions, which are directly or overtly related to the mathematical theme of the *Sulva* texts?

Our point is : *While there is a necessary connection between the technological problems and the mathematics in the Sulva texts, there is no such connection between this mathematics and the system of magico-religious beliefs.* As a matter of fact, the body of magico-religious beliefs is totally extrinsic to the mathematics of the *Sulva* texts. This is clear from a few obvious considerations.

First, the same mathematical problems would have remained in tact had the same brick-structures been required to serve some purpose other than the magico-religious ones. Thus, for example, a very important form of these brick-structures is required to have the shape of a falcon with specified size and made of a specific number of bricks arranged in a specified number of layers. According to the system of magico-religious beliefs, the use of such a structure in the sacrificial ritual ensures for the *yajamana* or rich patron financing the sacrifice the quick attainment of heaven. However, if we think of a rich patron wanting to have the same bird-like brick structure for some other purpose—say, as a decoration for his pleasure garden or for the play ground of his children—the technological requirements along with the collateral mathematical problems would have remained identical, though without being associated with the body of magico-religious beliefs.

Secondly, with all the importance seemingly attached to the brick-structures for magically ensuring the desired results, the priest-class recommending the construction of these tacitly admit that the same results could be magically ensured for the *yajamana* without the physical construction of the brick-structures. Thus, as we shall later see in some detail, the priests prescribing the rituals themselves claim that the results may as well be obtained with altars made only of spells (*chandas-citi*) or mind-made altars (*manomaya citi*) which can only mean imaginary altars as substitutes for physically constructed ones. Thus the actual or physical construction of the structures are really not so essential for the magico-religious belief-system as these are often thought to be. So also is the mathematical knowledge, which, however, is essential for the physical construction of the structures, irrespective of the circum-

stance of their serving magico-religious or any other secular purpose.

Pending a fuller discussion of the brick technology discussed in the *Sulva-sutra-s*, we quote here a few observations of Thibaut to have some idea of why a good deal of mathematical knowledge was essential for the physical construction of the structures, which, in priestly terminology, were called the altars—*citi-s* or *agni-s*. Discussing the more elaborate ones, he¹² observes :

Every one of these altars had to be constructed out of five layers of bricks, which reached together the height of the knee ; for some cases ten or fifteen layers and a correspondingly increased height of the altar were prescribed. Every layer in its turn was to consist of two hundred bricks so that the whole *agni* contained a thousand ; the first, third, and fifth layers were divided into two hundred parts in exactly the same manner ; a different division was adopted for the second and the fourth so that one brick was never lying upon another brick of the same size and form.

Secondly, the *Sulva-sutra-s* discuss not merely one type of brick-structure. The elaborate brick-structures discussed in these are supposed to have various shapes—those of the falcon with stretched wings, of the chariot wheel, of the triangular forepart of a cart (called *prauga*), of the tortoise, etc. Of these altars with various shapes, as Thibaut observes, the falcon-shaped one was the “most primitive”.¹³ For this was specified an area of seven and half square *purusa*, i.e. “the height of a man with uplifted arm. This rule was valid at least for the case of the *agni* being constructed for the first time ; on each subsequent occasion, the area had to be increased by one square *purusa*”.¹⁴ But this was not all. As Thibaut¹⁵ adds,

Now when for the attainment of some special purpose, one of the variations enumerated above was adopted instead of the primitive shape of the *agni*, the rules regulating the size of the altar did not cease to be valid. but the area of every *citi*, whatever its shape might be—falcon with curved wings, wheel, *prauga*, tortoise, etc.—had to be equal to $7\frac{1}{2}$ square *purusa-s*. On the other hand, when at the second construction of the altar one square *purusa* had to be

12. G. Thibaut, *On the Sulva Sutras*. Reprinted in *SHSI* II. 419.

13. *Ibid* II. 420.

14. *Ibid*. II. 420.

15. *Ibid*. II. 420-21.

added to the 7½ constituting the first *citi*, and when for the third construction two square *purusa*-s more were required for the shape of the whole, the relative proportions of the single parts had to remain unchanged. A look at the outline of the different *citi*-s is sufficient to show that all this could not be accomplished without a certain amount of geometrical knowledge. Squares had to be found which would be equal to two or more given squares, or equal to the difference of two given squares; oblongs had to be turned into squares and squares into oblongs; triangles had to be constructed equal to given squares or oblongs, and so on. The last task and not the least was that of finding a circle, the area of which might equal as closely as possible that of a given square.

To all this is to be added another point. Obviously enough, only one standardised brick-type with fixed shape and size could not meet the requirements of the construction of varied type of structures. In the *Sulva-sutra*-s, therefore, are specified how many varieties of bricks were required for the detailed demands for the construction of each type of the structure and we shall see into what meticulous details of calculations the texts had to go in order to be precise about the shape and size of each brick-type and about the mode of arranging these in different layers.

It is for meeting these essentially technological questions connected with brick-making and brick-laying that the *Sulva* mathematics came into being, notwithstanding the circumstance that in the Vedic text the brick-structures are said to be connected with a body of magico-religious beliefs—beliefs that are totally extrinsic to the *Sulva* mathematics. That is why we have observed that if we ignore or overlook the technology of brick-making and brick-laying of the *Sulva* texts, nothing substantial will be left in these having any mathematical interest, while it is possible to scrap the entire body of magico-religious beliefs from the texts without in the least affecting their mathematical contents.

Before we pass on to the nature of the mathematical contents of the *Sulva* texts however, we are confronted with a serious problem. If the *Sulva* mathematics is inconceivable without sophisticated brick technology, sophisticated brick technology is not easily conceivable in the period of the *Sulva* texts. In other words, from the archaeological point of view, we have here some kind of an anomaly: mathematics thriving essentially on sophisticated brick technology is found to be

embodied in texts of a period where there is no brick technology—not to speak of any sophisticated form of it.

5. AN APPARENT ARCHAEOLOGICAL ANOMALY

It is impossible, of course, to be exact about the date of *Sulva-sutra*-s. Depending on various circumstantial evidences, the modern scholars have proposed various possible dates for these works—or, more properly, the dates of the authorities with whose names the works are connected—Baudhayana, Apastamba, Katyayana, etc. These suggested dates range from 800 B.C. (Kane) to 250 B.C. (Keith), though of these 800 B.C. seems to be an exaggeration of the antiquity of the works just as 250 B.C. seems to be underestimating it. Without trying to enter into this chronological controversy, however, it may be permissible to assert that the actual date of the *Sulva* texts cannot fall outside the period intervening the two urbanizations, i.e. the “Dark Age” or “Dark Period” of the archaeologists.

This point is of material importance for our discussion, because an important feature of the period intervening the two urbanizations, as we have already seen, is the loss of the brick technology of the first urbanization. The re-introduction of brick technology—or, to be more specific, the technology of making and using burnt bricks—is generally admitted to begin again with what the archaeologists call the NBP period, i.e. the period the main index to which is a pottery-type called the Northern Black Polished Ware. This period, again, is roughly divided into two phases—the first dated 600-300 B.C. and the second 300-100 B.C. It is only in the second of these phases that we come across the re-introduction of brick technology in any noticeable scale. As R.S. Sharma summarily puts it, “while the second phase of the NBP is marked by . . . burnt brick structures, occasional tiles and ringwells, the first phase is marked by the absence of burnt brick structures and ringwells”.¹⁶

The introduction of NBP Ware is generally viewed as foreshadowing the Second Urbanization or the beginnings of early historical cities, a number of which are mentioned specially in early Pali literature. Here is how, in 1973, A. Ghosh¹⁷ sums up the archaeological evidences for the use of burnt bricks in

16. R. S. Sharma MCSFAI 91.

17. A. Ghosh CEHI 68-70.

connection with the formation of the cities in early historic India :

..of the use of burnt brick there is not much evidence in the earlier life of the cities. At most sites where evidence exists, burnt brick came into vogue either in the late phase of the Northern Black Polished Ware or, more commonly, in a still later period. Taking into consideration the metropolitan cities first, we find brick structures in the Northern Black Polished Ware levels, but it is not known from the published notices whether they were from the early or late levels : at Pataliputra, the origin itself of which being later than the advent of the Ware, it might be presumed that the structures belonged to a late phase ; Vaisali, where there was a single brick wall in pre-Sunga levels but in the Sunga level and onwards there was a network of brick structures : Ujjain, where there were mud, mud-brick and brick walls ; Besnagar (Vidisa) ; and Ahicchatra, where the use of burnt brick in the earlier period is attested, but where there was a free use of the material in the pre-Kushan, Kushan, and later levels.

Elsewhere we have explicit knowledge that burnt brick appeared only in the late phase of the Ware or even later. At Hastinapura, in Sub-period I of Period III, with that Ware, there were only two drains and a small wall, and in Sub-period III a long wall, followed by a large number of walls in Period IV, when the Ware had disappeared. At Rajghat, there were brick structures only in the late phase of the Ware, but in the next epoch there were a large number of structures. Mathura, with scanty burnt-brick remains in Sub-period I, had a vigorous building-activity in Sub-period III of Period II, both with the Ware. At Charsada, many of the early layers were associated with mud-brick and only the later ones with burnt brick. Comparable evidence is available at Tilaurakot, Atranjikhhera, Sonpur and Chirand and other sites. The evidence of Kausambi is no less significant : here too burnt-brick structures appear well after the introduction of the Northern Black Polished Ware.

Outside north India, at Navdatoli the first burnt-brick structure appears after 400 B.C. At Nasik, Nevasa and Tripuri the use of brick is post-Mauryan.

Evidence is thus complete that burnt brick became popular very well after the appearance of the Northern Black Polished Ware ; it became common only in the second century B.C. and abundant even later on. The early cities were contented with mud and mud-brick structures where stone was not available, with the possibility of wooden structures, the remains of which have not survived. It has been said : 'In India, till recently the existence of kiln-burnt brick houses distinguished the town from the village, and this could serve as a yardstick even in classifying older habitations'. [Y. D. Sharma] An application of this criterion would deny a civic status even to those places which were renowned cities at the time of Buddha.

More evidences are perhaps not necessary to show the absence of the technology of making and using burnt bricks—not at least in any significant form—after the decline or final disruption of the First Urbanization in c. 1750 B.C. and before considerable progress towards the Second Urbanization, in the later phase of the NBP Ware period in c. 300 B.C. But it may be useful to have here some clarifications.

Apart from a stray baked brick found in Sanghol¹⁸ and a fragmentary one found in Hastinapur¹⁹ or a broken one found in Abichchhatra²⁰—which are somehow associated with PGW culture but the evidences of which really prove nothing—G.R. Sharma, the excavator of Kausambi, claims to date the use of burnt brick in this site in 1025 B.C.,²¹ connecting it with the Harappans rather than the PGW people. As he²² puts his claim :

The early defences at Kausambi closely recall the Harappan citadel. The mud-packed rampart revetted externally with baked bricks in the so-called English bond in alternate courses of headers and stretchers, battered back to angles of 20° to 40°, bastions at intervals, rectangular towers and underground passage built on corbelled arch, are significant features of architecture at Kausambi with prototypes for each one of them in Harappan architecture. The very idea of town life was so far unknown in the Gangetic Valley. The defences show that in the first centuries of the first millennium B.C. Kausambi developed as a town fully equipped for its protection by the magnificent defences built on the Harappan pattern. Evidently, this was not an achievement of the P. G. Ware culture which shows a distinct aversion to the very concept of urban life in its earlier settlements in the Ghaggar Valley, the Punjab and Western U.P. Nor can it be associated with the Red Ochre-washed Ware. It is equally significant that P. G. Ware occurs at Kausambi two structural periods after the origi-

18. IAR, 1977-78. 43.

19. AI, No. 9-10, 17.

20. IAR, 1963-64, 43.

21. G. R. Sharma EK 22.

22. *Ibid.* 6. B. B. Lal (in HC, ed. Possehl, p. 336) comments : "Sharma's dating of the Kausambi's fortifications has been challenged by K. K. Sinha (1973) and A. Ghosh (1973). The grounds they have adduced against such an early chronology are quite valid and one would have expected the excavator of Kausambi to rethink the matter. Instead, he has come out with a renewed vigor about the Harappan influence on Kausambi."

nal construction of the defences. The recent discovery at Alamgirpura (District Meerut, U.P.) has established definite evidence of the penetration of the Harappan culture in the Ganga-Yamuna Doab: If the Harappans could reach the banks of the Hindon, a tributary of the Yamuna, the percolation and the survival of the Harappan influences at Kausambi only 300 miles down the Yamuna, is more than likely.

However, generally speaking, the view has not found favour with other serious archaeologists. The dating of Kausambi fortification by G.R. Sharma is rejected by K.K. Sinha²³, A. Ghosh²⁴ and others as too early. On the authority of Wheeler²⁵, R.S. Sharma argues that this date "cannot be pushed beyond 550 B.C. In fact the discovery of a cast copper coin may bring its date down to around 300 B.C."²⁶ B.B. Lal—besides rejecting G.R. Sharma's dating of the burnt bricks at Kausambi—vigorously argues against the possibility of any Harappan influence on the site.²⁷ In any case, the Kausambi excavation does not prove the technology of burnt bricks in the "Dark Age" intervening the two urbanizations. We shall later come to the question of G.R. Sharma's claim to unearth the ruins of an actual Vedic fire altar (*syenaciti*) at Kausambi.

But we cannot ignore or overlook in this connection the evidences unearthed by J.P. Joshi's explorations and excavations during the field seasons of 1975-76 and 1976-77 at Bhagwanpura (District Kuruksetra) and Dadheri (District Ludhiyana), revealing the use of burnt bricks. The excavations at Bhagwanpura, according to Joshi, "revealed a two-fold sequence of cultures designated as sub-Period IA and IB within a deposit of 2.70 m. showing for the first time that the Late Harappan Culture was interlocked with Painted Grey Ware Culture".²⁸ Describing the structures of Sub-Period IB, Joshi observes, "At first the people were living in round or semi-circular huts... In the next stage, the houses were built of mud walls... The third structural phase was associated with houses built of baked bricks of different sizes. Due to ploughing operation, all the

23. K. K. Sinha, in RIA, 1973. 231-38.

24. A. Ghosh, CEHI 81.

25. Wheeler EIP 130.

26. R. S. Sharma MCSFAI 59.

27. B. B. Lal, in Possehl (ed) HC, 336.

28. J. P. Joshi, in ME 1978, 98.

structures have been damaged. Whatever bricks were found *in situ* conformed to the following sizes : (i) $20 \times 12 \times 8$ cms. (ii) $12 \times 12 \times 8$ cms., (iii) $29 \times 22 \times 12\frac{1}{2}$ cms. (wedge shape), (iv) $20 \times 30 \times 8$ cms., (v) $16 \times 12 \times 4$ cms. Some of the bricks have deep finger marks".²⁹

Further :³⁰

At Dadheri, in a 6 m. cultural deposit a three-fold sequence of cultures was identified. Of these, the lowest, Sub-period IA, is represented by pure Late Harappan Culture, closely followed by Sub-period IB wherein Painted Grey Ware and Late Harappan pottery are found together.. In Sub-period IA evidence of mud-walled houses and huts is available. Other important finds include a huge storage jar (Pl. VIII) with late Harappan Painted and incised wavy lines of pre-Harappan tradition and late Harappan pottery of the usual type, copper objects, terracotta beads, wheels and round cakes, faience bangles and a terracotta painted bull.

In Sub-period, IB, Painted Grey Ware, black ware, grey ware, red ware and typical late Harappan pottery is available. In this Sub-period, three structural phases have been recognized. At first the people were living in semi-circular huts as attested to by the discovery of post holes. Three oval structures of burnt earth probably of religious character came from this phase. In the next stage, the houses were built of mud walls. One such room measuring 1.10×2.50 of a house complex has been noticed. The last phase is represented by a wall made of bricks, brickbats and brick jelly. Two sizes of burnt bricks, ($12 \times 12 \times 7$ cms., $25 \times 20 \times 5$ cms.) have been found. Other finds from this Sub-period include terracotta beads, copper ring, terracotta wheels and faience bangles. No Iron has been found.

The last phase of occupation of the site belongs to the medieval times. The finds from this Period II include remains of a mud wall, typical medieval plain and painted pottery and terracotta figurines and games-men.

Much, of course, remains to be clarified about the above. The dates assigned to the Bhagwanpura finds by Joshi range from 1500 to 1000 B.C.³¹; but we are not told about the dates assigned to the different Sub-Periods at the site. As R.S. Sharma rightly observes, "Only the publication of the full report can throw light on the stratigraphical position of these bricks which appear to be rather unusual if the PGW-

29. *Ibid.* 98-99.

30. *Ibid.* 99-100.

31. R. S. Sharma, MCSFAI, 23 ; see also p. 33 note 6.

iron phase is placed roughly in 1000—500 B.C. So far no Carbon-14 dates have been made available".³² Besides, B.B. Lal expresses strong doubts about the use of the expression of "Late Harappan" in the context of this site and comments: "However, the point to be emphasised is that the Bhagwanpura culture complex, composed of what can be termed an amalgam of the n -th generation of Harappans; the $n+x$ -th generation of pre-Harappans and the y -th generation of Harappan cousins, *was in no way urban*. It was in this essentially rural setting that the meeting with the PGW Culture took place, the period of overlap being termed IB at Bhagwanpura, and likewise at Dadheri".³³

In any case, from the point of view of what we are now discussing, namely brick technology, the discovery of some burnt bricks at Bhagwanpura and Dadheri, if assumed to belong to c. 1000 B.C., cannot but appear to be unexplained and an extremely odd phenomenon—as odd indeed as their sizes and proportions of their sides which answer neither to those of First Urbanization nor to any of the large variety of bricks we read in the Sulva texts. However, in default of an exact dating of the Bhagwanpura and Dadheri bricks and in the context of what is overwhelmingly obvious about archaeological data of the PGW sites in general, it may be permissible to work on the assumption that the technology of burnt bricks in any significant sense is absent throughout the period intervening the two urbanizations.

6. R. S. SHARMA'S THEORY OF MUD-BRICKS

We are thus confronted here with a serious problem. If—as we have already said and as we are later going to show in some detail—the mathematics embodied in the Sulva texts is inconceivable without the assumption of very sophisticated brick-technology, how are we to understand the fact that the Sulva texts themselves belong to a period in which, archaeologically speaking, the technology of making and using burnt bricks is conspicuous by its absence?

32. *Ibid*, 66 note 39.

33. B.B. Lal, in Possehl (ed.) HC 338.

R.S. Sharma seems to suggest a somewhat easy way out of the difficulty. His point is that the bricks mentioned in the Vedic texts—inclusive of the *Sulva-sutra-s*—need not be conceived as baked or burnt bricks at all. These were unbaked mud-bricks instead, which are easily conceivable in the PGW sites that are usually viewed as Vedic settlements, and hence the making of these is easily conceivable as forming part of the technology known to the Vedic people. As he³⁴ observed:

The PGW mud-brick walls found at Hastinapur remind us of later Vedic references to bricks in connection with the construction of altars; seven brick names are found in the *Taittiriya Samhita*, nine in the *Kathaka Samhita*, and eleven in the *Maitrayani Samhita*. In the *agnicayana*, the stacking of the bricks for the fire altars which is made obligatory in the *mahavrata* and optional in other *soma* sacrifices, the building of the *uttaravedi* involves five courses of bricks, making 10,800 bricks in all, in prescribed patterns often in the form of a bird with outstretched wings. But generally the PGW sites, except at Bhagwanpura and a few other places where the fire burnt bricks have been reported but not accounted for, do not yield fire-baked bricks; similarly the later Vedic texts do not know of these. Of course a battered facing of brick on the mud ramparts of Kausambi has been discovered, but it cannot be pushed beyond 550 B.C. In fact the discovery of a cast copper coin may bring its date down to around 300 B.C. Therefore, the bricks mentioned in the Vedic texts were not generally baked in fire. A potter's kiln of the PGW level has been discovered in Atranjikhhera. Such a kiln is known by *apaka* (Hindi *ava*) in the Vedic texts, but no term for brick-kiln is found in Vedic sources. *The old Vedic practice of using unbaked bricks for religious purposes continues* in Maharashtra and possibly in the other parts of the country. The total picture of PGW settlements does not warrant their characterization as urban, as has been done by Wheeler; at best they can be called proto-urban towards the end of the PGW period. The later Vedic texts do not know of urban life. Kampila, the capital of Pancalā, may have been an administrative settlement. The term *nagara* occurs in an Aranyaka and *nagarin* in two Brahmanas which are not earlier than 600 B.C.

R.S. Sharma does not mention here the *Sulva-sutra-s* or the mathematics embodied in these. This is perhaps because of his view already mentioned that the mathematics embodied in the *Sulva* texts arose out of the administrative requirements of land-measurement—a view evidently originating from the observation of Herodotus but rejected by archaeological-technological

34. R. S. Sharma, MCSFAI 58-59. Emphasis added.

considerations mentioned by Gordon Childe and J.D. Bernal. However, the point is that Sharma discusses here the question of *agnicayana* or the construction of the fire altars with bricks and though the Yajurvedic texts like *Taittiriya-samhita* and even the *Brahmana*-s enable us to understand the magico-religious beliefs imputed to these brick-structures, the *Sulva-sutra*-s are for us the most essential texts for understanding the technique for the physical construction of such structures. Hence is the obvious difference in the view of the bricks taken in the *Taittiriya-samhita* etc. and the *Sulva* texts—a difference which we shall later discuss. For the present let us concentrate on the other point stressed by R.S. Sharma in his observation just quoted, namely that the *Vedic texts refer to only mud-bricks and not burnt-bricks* because from the archaeological view-point only the former is to be expected in the PGW sites usually associated with the Vedic peoples.

But is it a fact that the Vedic texts speak only of mud-bricks and are unaware of burnt bricks? The answer is evidently in the negative.

Already in the *Satapatha Brahmana*³⁵ we read :

Having gathered both that clay and water, he made a brick : hence a brick consists of these two, clay and water. He considered, 'Surely if I fit this (matter) such as it is unto my own self, I shall become a mortal carcase, not freed from evil : well then, I will bake it by means of fire'. So saying, he baked it by means of the fire, and thereby made it immortal.. Hence they bake the bricks with fire ; they thereby make them immortal.

The *Baudhayana Sulva-sutra*, too, while discouraging the use of over-burnt bricks (ii.55) and also by advising how to make up for "that which is lost by the heat and the burning from the right size of the bricks" (ii.60), is indicative of burnt rather than simple mud-bricks. Here again the word used is *paku* which means firing. For unbaked brick the word would have been *ama*.

7. BURROW ON 'ARMA' AND 'ARMAKA'

So the anomaly remains. We do come across references to burnt bricks in the literature of a period in which—archaeo-

35. *Satapatha Brahmana*, vi. 2-1.8-9. The text, by clearly using the expressions *agnina pacani* and *aganina apacat* leaves absolutely no scope for doubt that the bricks spoken of were burnt in fire.

logically speaking—the technology of making and using these cannot be admitted. How are we to account for it?

At the present stage of research, it is evidently premature to expect a full answer to this question. What is possible, nevertheless, is to raise some counter-questions that may perhaps be pointers to further research.

The first question that occurs to us in this connection is : Could it be that the Vedic peoples, though without the know-how of making and using burnt bricks, were acquainted with ready-made bricks, i.e. with bricks made and used by others centuries before, or to be more specific, during the period of the First Urbanization, when highly sophisticated brick-technology was an accomplished fact, and when, therefore, the possibility of the emergence of mathematics from this technology cannot be *prima facie* impossible? If there be anything in such a possibility, the presumption would be that the roots of the mathematics of the *Sulva-sutra*-s are to be traced back to the Harappan culture, though we are yet to know how it was transmitted and codified in the *Sulva* texts many centuries later.

The primary evidences for the making of mathematics in Harappan culture are no doubt to be searched from the archaeological data. Before passing on to these, however, we may try to review some circumstantial evidences of the later period indicating the general possibility of the Vedic people being acquainted with ready-made bricks of the Harappan period.

We begin with the brief but exceedingly interesting article by T. Burrow *On the Significance of the Term arma-armaka in Early Sanskrit literature*.³⁶ The article needs to be read in full, though we have the scope here to mention only some of its salient points.

Though the word *arma* fell into disuse in classical Sanskrit literature, Burrow draws our attention to its use in Panini's grammar and the *Kasika* commentary on it as illustrating rules "concerning the accentuation of certain compounds having this word as last member". This list from the grammatical literature given by Burrow includes Bhutarma, Adhikarma, Sanjivarma, Madrarma, etc. On the authority of V.S. Agarwala³⁷, Burrow observes, "All these are place names and the

36. Burrow in *JIH*, Vol. 41. 1963, 159-166.

37. V. S. Agrawala, *India as known to Panini*, 66-67.

element arma- at the end means a ruined site or settlement".³⁸ The meaning 'ruined site or settlement' is taken from the commentators who use the expression *vinastagrama*—, which in later times literally means 'ruined village'—*grama* meaning village, while the word for city is *nagara*. But on the authority of Agrawala, again, Burrow shows that, in the context of its use in *Panini*, it should rather mean ruined cities³⁹ :

As V. S. Agrawala points out, the two terms *grama* and *nagara* were used indiscriminately in the Vahika country (i.e. the Punjab), although distinguished in Eastern India as 'village' and 'town'. In the case of these place names one would expect that the meaning required would be 'town' or 'city' rather than 'village', since any material remains of mere villages would be insignificant and would not qualify for a name having general currency and for that reason being worthy of mention in the grammatical work of Panini. To account for this we must assume that they were substantial ruins, and that the word *grama* in this context is to be taken in the sense in which it was used in the Vahika country. From this one might be tempted to think that the situation of these ruined sites was in fact in the Vahika country. Certainly the only one that can be localised from the name—Madrarma—does not (*sic*) belong to this region, and it would not be surprising if the same applied to the other names formed on this pattern, particularly when we bear in mind the fact that in Panini's geography there is a distinct bias towards the North-west.

Where, then, were these ruined cities? Could these be earlier Aryan settlements or the ruined cities of the ancient Indus civilization, which, in the Vedic period, were presumably quite abundant in the North-west? Burrow⁴⁰ answers :

It is to be remembered that in Panini's time (say fourth century B.C.) the period of urban civilization for Aryan India was of comparatively recent origin so that although one might expect a few such deserted or destroyed cities (and presumably Navarma- was one of these), one would not expect them to be so thick on the ground as this large collection of names would indicate. On the other hand the number of Indus sites of the required magnitude already identified in this region is sufficient to explain this long string of names (which is to be taken as typical and not exhaustive), and it is to be remembered that during the early Aryan period the ruins of many Indus cities must have formed a conspicuous feature of the countryside.

38. Burrow, *op. cit.* p. 159.

39. *Ibid.* 159-60.

40. *Ibid.* 160.

This leads Burrow to search for evidences of the use of the word *arma* (or its derivative *armaka*) in literature earlier than Panini. And he comes across a number of these in the Vedic literature, some of which appear to have much significance. Here are a few of these.

The *Latyayana Srautasutra* X. 18.3 says : "On the Sarasvati. there are ruined sites called Naitandhava ; Vyarna is one of these". Other Vedic texts like *Apastamba Srautasutra* (xxiii. 13.12), *Sankhyayana Srauta-sutra* (xii. 29.28), *Pancavimsa Brahmana* (xxv. 13.1) also mention Naitandhava and Vyarna, though the one first quoted is very significant. As Burrow observes, "The mention of these ruined sites with the precise information about their location, informing us that they were situated along the Sarasvati, is exceedingly valuable information, since it is now well established that the Indus sites are a feature of this region. A recent excavation of one of these sites, at Kalibangan on the south side of the Ghaggar (ancient Sarasvati) has demonstrated the importance of this region as a centre of the Indus civilization".⁴¹ The *Latyayana Srautasutra* (x.19.9) speaks also of ruined site (*armaka*) along the right bank of the Drsadvati—a location of Harappan ruins again. Other references to *arma* and *armaka* in the Vedic literature, according to Burrow's argument, indicate the destruction or devastation of the cities of the Indus civilization by the invading Vedic Aryans—a view about which there is much debate in recent times and to which we shall later return. For the present our point is that Burrow's interpretation of *arma* (and *armaka*) as ruined Indus sites seems to be of much importance for what we are going to argue. Let us see why it is so.

Burrow⁴² observes,

A compound *arma-kapala* meaning 'a tile from a ruined site' occurs not infrequently in the *Srautasutras* (e.g. *Baudhayana* ix.1.3, etc.) where it appears among a list of paraphernalia for a sacrifice. In this connection the *Vadhulasutra* glosses : *atha yad armakapalani bhavanti armad evainam tat prthivyah sambharati*—'since there are tiles from a ruined site, in this respect he assembles it (the fireplace) from a ruined site of the earth.' From these *sutra* references we

41. *Ibid.* 162.

42. *Ibid.* 161.

gather that *arma-s* or ruined sites were a commonplace thing in the Vedic period, since these *arma-kapalani* prescribed in the ritual appear to have been readily available. This is in agreement with the fact that material remains of the Indus Civilization have been located in abundance throughout the territory occupied by the Vedic Indians subsequent to its downfall.

The 'paraphernalla for a sacrifice' referred to above are the special type of earthen vessel prescribed for the ritual use, a fuller discussion of which we have in the article *Vedic Literature on Pottery* by Shivaji Singh.⁴³ In this article Singh mentions the priestly instructions concerning the preparation of clay for the making of the earthen vessels to be used in the Vedic sacrifice. According to the *Taittiriya-samhita*, as Singh observes, "potsherds collected from ancient deserted sites (*arma-kapala*), sand (*sarkara*) and hairs (*ajaloma* and *krsnajinaloma*) were to be mixed with clay".⁴⁴ In the notes, Singh adds that Sayana, commenting on the Brahmana portion of *Taittiriya-samhita* iv.1 explains *arma-kapala* as (potsherds from) *cirakala-sunya-grame bhumau avasthitani puratanani*, i.e. ancient (potsherds) existing in the eternally deserted cities. (We have already noted Burrow's argument why in this context the word *grama* should preferably be taken to mean 'city' rather than 'village'). Since, obviously enough, no city can be "eternally deserted", we have to take Sayana here as referring to cities that remained deserted from a very ancient period. As far as our present knowledge goes, only the ancient Harappan ruins can answer to what is meant by Sayana.

To sum up the discussion so far : From the ancient Vedic literature to Panini we come across references to ruined cities as *arma*, which, moreover, were presumably quite commonplace in north-west India. Secondly, according to the ritual instructions, potsherds from these ruined cities had to be collected for the preparation of clay, from which to fashion earthen vessels for Vedic sacrifices.

To these we have to add here only one point. Archaeologists have no doubt found plenty of potsherds from the ruined Harappan sites. However, if anything is much more conspicuous about the ruined cities like Harappa, Mohenjodaro and Kalibangan it is the heap of burnt bricks. Even after

43. See B. P. Sinha (ed) PAI 301-73.

44. *Ibid.* 307.

the brick-robbery in massive scale and using these as ballasts for the railway lines from Lahore to Multan and from Hanu-mangarh to Suratgarh, the burnt bricks still surviving in the ruined sites of the Indus Valley civilization are most imposing. If, therefore, the Vedic priests were actually collecting pots-herds from these ruined sites, it is not difficult to conceive how they could speak of burnt bricks without acquiring the technology of making and using these.

Do we have here a clue to the apparent anomaly of the references to burnt bricks in the literature of a period which, archaeologically speaking, is unaware of the know-how of making and using burnt bricks?

Not that such a supposition is free from difficulties. We do not have in the Vedic literature any reference to the collection of *arma-istaka* or bricks from ruined sites as we have to that of *arma-kapala*-s. Nor have we any direct archaeological evidence of the re-use of ready-made bricks for the construction of the Vedic sacrificial altars. The earliest reference to the fire altars in Vedic literature are to be found in the *Yajurveda*—particularly the *Taittiriya-samhita*. However, as we shall presently see, the text is much too interested in describing the mysterious magical efficacy of the bricks to give us any physical description of these—descriptions which could have enabled us to compare these bricks with the Harappan ones. Nor is it possible for us to make much of the quaint brick-names of the *Taittiriya-samhita*, some idea of which we shall presently have. On the whole, it may not be an error to think that the *Taittiriya-samhita*, viewing as it does the bricks as highly mysterious entities with wonderful magical potency seems to suggest that the priests in the *Taittiriya-samhita* knew of bricks without knowing what these really were.

Things would have been helpful for us had the archaeologists been able to discover any actual ruin of a sacrificial altar belonging to the Yajurvedic period. But the fact is that nothing like that is so far discovered. The earliest archaeological evidence of what is claimed to have been a Vedic fire-altar—a *Syenaciti*—comes from the excavation of Kausambi. According to G.R. Sharma, the excavator of Kausambi, the site has revealed not only the remains of a brick built *Syenaciti* but also other relics like animal and human bones reminiscent

of the ritual prescriptions of the Vedic priests.⁴⁵ But he has not tried to correlate the brick-sizes in the ruins of this "altar" with what are mentioned in the *Sulva* texts regarding the construction of the *Syenaciti*. The average brick-sizes mentioned by G.R. Sharma as having been generally used in Kausambi, namely $19.5 \times 13 \times 2.75$ inches⁴⁶ answer neither to the known brick-sizes of the Harappan ruins nor to those prescribed in the *Sulva* texts. In any case, even admitting his claim that the structure unearthed represents the remains of a *Syenaciti*⁴⁷, its date cannot be pushed back to the Yajurvedic period. As Y.D. Sharma observes, "The sacrifice is believed to have been performed by the founder of the Mitra dynasty whose coins have been recovered in abundance from corresponding levels"⁴⁸. The coins of Mitras could not be earlier than the second century B.C.⁴⁹. Other archaeological evidences suggesting Vedic fire-altar are much later. One of these, e.g., come from the excavation of Jagatram, about 30 miles to the North-west of Dhera-dun, and is dated about the third century A.D.⁵⁰

Such, then, is our present knowledge of the archaeological evidences about the Vedic fire altars and it is no use speculating on what might have eluded the archaeologists' spade so far. Nevertheless, the fact remains that we have in the late Vedic literature unmistakable evidence of mathematics emerging from the requirements of brick technology while the authors of this literature—without any knowledge of the technology in their times—could conceivably have any acquaintance with it only in the ruined Harappan sites. Could this be a pointer to the possibility of the whole thing—the brick technology as well as the mathematics emerging to meet its requirements—

45. G. R. Sharma, EK, pp. 87-206.

46. *Ibid.* 27.

47. Lal B.B, personal correspondence dated 17th July, 1985, observes : "any way, my considered view is that the brick assemblage concerned is not at all a *Syenaciti*. It represents the collapse of an adjacent brick-wall pertaining to the fortifications." We are expecting the publication of his detailed discussion of the point in the *Puratattva*.

48. Y.D. Sharma, ARMM. I. 55.

49. Bela Lahiri, 90.

50. Excavated by T.N. Ramachandran, I.A. 1953-54, 10-11, See also B.B. Lal in CF Dec. 1961, 36.

actually developed in Harappan culture and somehow transmitted to the Vedic peoples of later times? It is premature at the present stage of our knowledge to try to arrive at any categorical answer to this question. What is possible—and perhaps also necessary—is to note some relevant points which may stimulate further research.

8. EVIDENCE OF THE SATAPATHA BRAHMANA

After the *Taittiriya Samhita* the question of Agnicayana or that of the ritual building of the fire-altar is elaborately discussed in the *Satapatha Brahmana*. As contrasted with the *Taittiriya Samhita*, in which the bricks are viewed as some mysterious entities with quaint names and quaint magical potency, the *Satapatha Brahmana* takes on the whole a comparatively realistic view of these, though of course without scrapping the associated ancient magico-religious beliefs. It is only in the *Sulva* texts that we come across a well-defined technological view of the bricks, specifying their exact shape, size, etc. We shall later return to some details of these. For the present, we have another interesting point to note about the *Satapatha Brahmana*.

Like the other *Brahmana*-s, the *Satapatha*, too, comes down to us as appended to one recension of the *Yajurveda*. In the *Yajurvedic* text there is constant mingling of magical formulas with explanatory portions of which only the latter is strictly called the *Brahmana*-s. The class of priests called the *Adhvaryus* wanted entirely to separate exegetic matter from the magical formulas or spells proper. The name given to the school of *Adhvaryus* responsible for the preparation of the *Satapatha Brahmana* is *Vajasaneyins*, its origin being ascribed to one *Yajnavalkya Vajasaneya*. "The *Brahmana* of the *Vajasaneyins* bears the name *Satapatha* i.e. the *Brahmana* of 'a hundred paths', because it consists of a hundred lectures".⁵¹

We have noted it mainly to emphasise one point. *Yajnavalkya* is expected to be the authority for the theological discussions in the text. But the text in the form in which it reaches us does not satisfy the expectation. We quote *Eggeling*⁵² at some length who draws our attention to this :

51. *Eggeling* in *SBE* vol. xii. Intro. pp. xxvii-xxviii.

52. *Ibid* Intro. xxxi.

As regards the earlier portion of the work, however, it is a remarkable fact that, while in the first five books Yajnavalkya's opinion is frequently recorded as authoritative, he is not once mentioned in the four succeeding *kandas*. The teacher whose opinion is most frequently referred to in these books is Sandilya. This disagreement in respect of doctrinal authorities, coupled with unmistakable differences, stylistic as well as geographical and mythological, can scarcely be accounted for otherwise than by the assumption of a difference of authorship or original redaction. Now the subject with which these four *kandas* are chiefly concerned, is the *agnicayana* or construction of the sacred fire-altar. For reasons urged by Professor Weber, it would appear not improbable that this part of the ceremonial was specially cultivated in the north-western districts; and since the geographical allusions in these four *kandas* chiefly point to that part of India, while those of the other books refer almost exclusively to the regions along the Ganges and Jumna, we may infer from this that the fire-ritual adopted by the Vajasaneyins at the time of the first redaction of their texts—that is, of the first nine *kandas*, as far as the Brahmana is concerned—had been settled in the north-west of India.

Here, however, we meet with another difficulty. The tenth book, or *Agnirahasya*, deals with the same subject as the preceding four *kandas*; and here also Sandilya figures as the chief authority, while no mention is made of Yajnavalkya.

What concerns our present discussion is not the question of the actual redaction of the *Satapatha Brahmana* in its present form. But the pointer to north-west India as the region from which it appears to incorporate within itself the matters concerning the *Agnicayana* or the ritual building of the fire-altar—and therefore by implication also of the brick-technology required for the altar construction—can by no means be ignored or overlooked. In fact, it acquires much more importance for us today than it could possibly have during the time of Weber (1805-1901) on whose discussion⁵³ of the geographical allusions in those portions of the *Satapatha Brahmana* Eggeling depends. The same is true of Eggeling's comment just quoted which was first published in 1882. Nothing whatsoever was known in their times of the imposing civilization in north-west India with its remarkably developed brick technology. With the discovery of the Indus Valley civilization, however, the peculiarities already noted by Weber and Eggeling cannot but lead us to a new question. Could it be that the brick-structures

53. A Weber, *Ind. Stud.* I., 187 seq. and XIII., 266ff.

spoken of in the *Satapatha Brahmana* are indicative of the heritage of the Harappan culture? The recent discovery of some structures generally considered as "fire altars" specially at Kalibangan⁵⁴ and Lothal⁵⁵ apparently strengthens the possibility, though it remains for us to see the hazards about any hasty conclusions about these.

9. THE QUESTION REOPENED

With the dramatic expansion in our understanding of the ancient Indian history by the recent archaeological work, the entire question of the Agnicayana is reopened by a section of the modern scholars. Some of them are trying to argue that the ritual was borrowed by the Vedic people from the Harappans. The question of the ritual as such is, of course, outside the scope of our own discussion. Nevertheless, some of the recent views expressed do interest us, because the question of the Agnicayana is inextricably connected with brick technology. As a matter of fact the vital dependence of this ritual on brick technology is used by some scholars as an evidence of the ritual itself having been a Harappan survival.

As Hyla Stuntz Converse⁵⁶, in her article *The Agnicayana Rite : Indigenous Origin ?* very strongly argues :

The question of brick is of major importance. The Harappa civilization, whose last, flood-damaged strongholds in the north were overthrown by the invading Aryans in battles commemorated in the *Rgveda*, was a brick-using culture. The Harappans used millions of kiln-fired bricks as well as countless sun-baked ones.. The bricks of the Harappa civilization in its mature phase were beautifully made, well fired, and standardized in size..

Now, in the whole of the *Rgveda* there is no word for brick, nor any descriptive phrase for bricks. So far no ruins of brick dwelling have been found that can be attributed to the Aryans in the early *Rg-Vedic* period.. There are also no references to bricks in the *Rg-Veda Brahmanas* and outside of the Agnicayana sections of the *Samhitas* and *Brahmanas* of the *Yajurveda* tradition, no significant reference to bricks occur in these or in the *Samaveda Brahmanas*. Thus, in the *Brahmanas*, when references to brick begin to appear, their use is confined to one specialized rite, and the rite itself is found only

54. Allchins RCIP 183 and 303.

55. S.R. Rao, LIC 139-40.

56. H.S. Converse, in HR Vol. xiv, No. 2, 83-84.

in the Yajurveda tradition. The fire altars in the other rites were made of packed earth, not bricks.

The size of the bricks to be used in the rite was one foot square, and half-bricks were also to be used (SB vii. 5.3.2; viii. 7.2.17). This size and shape corresponds very closely to that of the Harappa bricks described above. The lack of any bricks in the early Vedic tradition and the presence of bricks in large numbers and of the same size in the adjacent indigenous Black-and-Red Ware territory suggest that the Black-and-Red Ware culture is the source of the Agnicayana brick-making skills.

It is not necessary for our present purpose to enter into the technicalities of the Black-and-Red Ware pottery, which Converse readily takes as "the distinctive trait of the indigenous non-Vedic culture". Other archaeologists do not agree to this. A. Ghosh, for example, wants to see the Black-and-Red Ware in the expression *nilalohita* of the *Atharvaveda*.⁵⁷ Though Converse herself for her basic argument wants to depend much on the assumption of the Black-and-Red Ware being non-Vedic or pre-Vedic, the main drive of her argument is not basically affected by it, because she offers other evidences for the possible Harappan origin of the Agnicayana, the main point of which remains unaffected by what the archaeologists may finally conclude about the Black-and-Red Ware.⁵⁸ The most important of these arguments seems to be that Agnicayana is not conceivable without brick technology, "although the Vedic Aryans were not a brick-making people."

Frits Staal, in his *The Ignorant Brahmin of the Agnicayana*,—in which he argues that the apparently quaint concept of the "ignorant" priest occurring in the Vedic literature makes "sense only if he is a representative of a pre-Vedic fire cult"—finds it necessary to come back to the evidence of the brick technology. After reviewing the literary evidences which he considers important for his main arguments, he⁵⁹ adds :

If we wish to understand more, we have to go beyond the texts and place the Agnicayana in a wider historical perspective. The techniques for firing bricks, which we meet for the first time in Yajurvedic texts dealing with the Agnicayana, could not have been im-

57. A. Ghosh, CEHI 6.

58. H.N. Sing, "Black-and-Red Ware" in EIP 267-281.

59. F. Staal, in ABORI 1978, 345.

ported by the Vedic nomads who had earlier entered the subcontinent from the northwest. Nomads have no need for bricks. The bricks, in fact, occupy an exceptional position in the Vedic ritual, when all implements are made of perishable materials, and are taken away, destroyed, burnt or immersed in water after a ritual performance has been completed.

For the firing of bricks we have to look in a non-nomadic direction—i.e., for a sedentary civilization. The fact which springs to mind is that in the Harappa civilization the use of fired bricks was normal and widespread. We need not jump to the conclusion that the Agnicayana was a Harappa ritual, though Converse (1974) has certainly forwarded weighty arguments (together with some incorrect ones) in support of such a hypothesis. Whatever else may be true, it is certainly reasonable to suppose that knowledge of the techniques for firing bricks was preserved among the inhabitants of the subcontinent even after the Harappa civilization had disappeared. The Vedic nomads, who by the time of the *Yajurveda* were in close interaction with the indigenous population and had intermarried and accepted many of them within their fold, adopted these techniques in the construction of the Agnicayana altar.

Whether the "technique for firing bricks" is already to be found in the *Yajurveda* is, of course, a very doubtful point. Neither the internal evidences of the *Yajurveda* nor the archaeological data we have of the period of the formation of the text point to such a possibility though the *Satapatha Brahmana* clearly speaks of burning the bricks. We have already seen, how—without this technology—the Yajurvedic priests could possibly speak of the *istaka*-s. On this point, therefore, Staal's observations may be in need of some modification.

C. G. Kashikar critically reviews the opinions of both Converse and Staal. Mainly on theological grounds he argues that the Agnicayana could as well be "an extension of Vedic Aryan rituals". Also the apparently quaint expression "ignorant Brahmin," Kashikar argues, can have an alternative explanation, mainly from the theological viewpoint again. What concerns our own discussion, however, is not theology but brick technology. On this point Kashikar⁶⁰ observes :

It is true the *Rgveda* does not contain any reference to bricks, even though the existence of some kind of pottery can be thought of in the Rgvedic period. The *Brahmanas* of the *Rgveda* and *Samaveda* cannot be expected to contain any reference to bricks because the

60. C.G. Kashikar, in ABORI, 1981. 125.

executive function of the piling up of the altar was outside their scope. Even in the *mantra* and *brahmana* portions of the *Yajurveda* one cannot expect any reference to brick outside the piling up of the fire-altar. It is obvious that the Agnicayana rite was introduced in the Vedic rituals after the Vedic people became conversant with the use of bricks. As already noted, Vedic Aryans came into close contact with the Dravidians and other people in the north-west region of India even in the days of the *Rgveda*, and the process of cultural give-and-take had already started. The racial admixture, at least to a certain extent, was a natural consequence. The Vedic people, characterised as a racial complex, might naturally be expected to have adopted a number of worldly things from the indigenous people, among which the use of fired bricks for housing purposes must have been included.

But the only reference Kashikar gives from the Vedic ritual literature for the use of bricks for housing purposes is from Kesava's *Paddhati*, a 13th century commentary on the *Srautasutra*-s.⁶¹ The text is obviously too late to prove his thesis. It seems, thus, that after accepting bricks originally for ritual purposes, the Vedic people had to wait long for the technology to be used for house construction.

More interesting, however, is Kashikar's argument against Converse, who tries to correlate the brick-sizes of the Harappan cities with these mentioned in the *Satapatha Brahmana*. Argues Kashikar⁶² :

The comparison would hardly serve any purpose because the *citi*, even following the *Satapatha Brahmana*, would require bricks of various sizes and patterns. Even though the *Satapatha Brahmana* directly mentions only a few of them, the other sizes mentioned in the *Katyayana Srautasutra* and the *Istakapurana Parisista* need to be presumed even in the case of *Satapatha Brahmana*. Thus square bricks with each side measuring 24, 18, 12 or 6 *angulas* are mentioned. Besides these, oblong bricks of various sizes were also required. There are other varieties of the altar which need bricks of acute and obtuse angles and also curved bricks.

But all this really brings us back to the point we have been trying to drive at. Already the *Satapatha Brahmana* presupposes a long and sophisticated tradition of brick technology, while—archaeologically speaking—there is no such tradition in the immediate context of the text. On the contrary, there is only

61. *Ibid* 126, note 13.

62 *Ibid* 124-125.

one tradition from which the ritual texts like the *Satapatha Brahmana* could possibly borrow or assimilate such sophisticated brick technology. And that is the tradition of the Mature Harappan Culture.

10. PREHISTORY OF SULVA GEOMETRY

But let us return to the question of geometry and the *Sulva* texts.

The *Sulva-sutra*-s introduce us to a different technological climate. The bricks are no longer just mysterious entities with quaint magical potency, as these basically appear to be in the *Yajurveda*. Though without questioning the Yajurvedic priests and even accepting their dictations, the *Sulva-sutra*-s in fact take a different view of these. The bricks are deliberately made according to certain specific shapes and sizes ; these are deliberately dried and burnt. Attempt is made to determine exactly how much in size and area these lose as a result of drying and firing,⁶³ so that provision may be kept in the original mud brick for this shrinkage and eventually burnt brick is obtained according to their exact area required for the altar-making. Care is taken to see that the bricks are neither overburnt nor underburnt⁶⁴ and thereby become useless for construction purposes. In short, brick technology is discussed in the *Sulva-sutra*-s not from the standpoint of any outsider marvelling at the bricks but from that of the technologists themselves. More than this. The *Sulva-sutra*-s do not merely discuss the technology of *making* bricks ; the texts are as a matter of fact much more interested in the technique of *using* these bricks for the construction of certain pre-conceived structures, however complicated these might have been. However, what is most remarkable about the *Sulva-sutra*-s is that these are not mere manuals or handbooks for the craftsmen and the technicians ; these indicate also the awareness of developing the exact theoretical propositions which, as actually developed in the *Sulva-sutra*-s, come down to us as the earliest documents of geometry in ancient India.

The standardization of the theoretical knowledge in the form of geometrical propositions, along with the development of the terminologies required for these, evidently presuppose a pro-

63. *Baudh. Ss* ii 60.

64. *Baudh. Ss* ii. 55.

longed period. Since all this is the outcome of the techniques of brick making and brick laying, the formation of the *Sulva* geometry presupposes a long and sophisticated tradition of brick technology. However, if we are to place the *Sulva-sutra-s* roughly in the fifth or fourth century B.C.—as the modern scholars propose to do—we find practically nothing in the immediate historical context of these texts to suggest any long and sophisticated tradition in brick technology. How, then, are we to understand the formation of the *Sulva* geometry?

B. B. Datta concedes to the possibility of the *Sulva* geometry having some anterior stage of development. He speaks of “the growth and development of the Hindu Geometry from its earliest state down to the one in which we find it now in the *Sulba*”.⁶⁵ Thibaut also suggests that the *Sulva* geometry necessarily presupposes a long time of development. As he puts it, “Regarding the time in which the *Sulva-sutra-s* may have been composed, it is impossible to give more accurate information than we are able to give about the date of the *Kalpa-sutra-s*. But whatever the period may have been during which the *Kalpa-sutra-s* and *Sulva-sutra-s* were composed in the form we have now before us, we must keep in view that they only give a systematically arranged description of sacrificial rites which had been practised during long preceding ages”.⁶⁶ But where are we to look for the “long preceding ages”, the formative period of the *Sulva* geometry? Working as he did in the last quarter of the nineteenth century, Thibaut himself had nowhere else to look for it than in the earlier strata of the Vedic literature itself. And he fails to find there anything actually foreshadowing the *Sulva* geometry. The fact is that as we move backward from the *Sulva-sutra-s* to the earlier strata of the Vedic literature, we are confronted more and more with the mystery-mongering about the bricks and fire altars rather than either with the technology of actually building these or with the geometrical calculations required by the technology. Thibaut, therefore, wanted somehow to imagine that the formation must have had a pre-history among the Vedic priests themselves—the *Adhvaryus*—though without knowing what it actually had been. As he puts it, “The rules for the size of

65. B.B. Datta SS 20.

66. Thibaut, in SHSI, ii. 472.

the various Veda-s, for the primitive shape and the variations of the *agni* etc. are given by the *Brahmana*-s, although we cannot expect from this class of writings explanations of the manner in which the manifold measurements and transformations had to be managed. Many of the rules which we find now in *Baudhayana*, *Apastamba* and *Katyayana*, expressed in the same or almost the same words, must have formed the common property of all Adhvaryu-s long before they were embodied in the *Kalpa-sutra*-s which have come down to us".⁶⁷

Writing on the *Sulva-sutra* in 1875, as Thibaut did, what else could he do than try to convince himself that the *Sulva* geometry must have had its origin somewhere in the earlier stage of the tradition of the Vedic priests themselves, though without at all knowing where it could be.

67. *Ibid.*